Comment on "Adiabatic domain wall motion and Landau-Lifshitz damping"

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A recent article by Stiles *et al.* [Phys. Rev. B **75**, 214423 (2007)] has argued, using analysis of spin-transfer torque driven domain wall motion in magnetic nanowires, that the original Landau-Lifshitz form of phenomenological damping in the micromagnetic equations of motion provides a more physically sensible interpretation than that of the Gilbert damping form, and specifically claims that only Landau-Lifshitz damping uniquely maintains the physically intuitive damping property of always reducing magnetic free energy with spin-transfer torques present. The present article considers a more general, energy-based comparison of these two sets of equations when any nonconservative (e.g., spin-transfer) torques are present. It is instead concluded here that Gilbert damping provides the more physically sensible model over the form of Landau-Lifshitz argued in favor of by Stiles *et al.*, and that of these two only Gilbert damping guarantees a negative-definite contribution to changes in magnetic free energy.

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I. INTRODUCTION

The recent work of Stiles *et al.*[1] reconsiders a comparison of the well-known Landau-Lifshitz[2] (LL) and Gilbert[3] (G) equations of motion for the magnetization in (transition-metal) ferromagnets in terms of their predictions regarding spin-transfer torque induced domain wall motion in ferromagnetic nanowires. In *conservative* systems for which the effective field, $\boldsymbol{H}_{\text{eff}}$, is completely defined in terms of the free-energy functional, $E(\hat{\boldsymbol{m}})$, for the magnetic system, the agreed upon form of the LL and G equations is

$$\text{G: } \frac{d\hat{\boldsymbol{m}}}{dt} = -\gamma(\hat{\boldsymbol{m}} \times \boldsymbol{H}_{\text{eff}}) + \alpha\left(\hat{\boldsymbol{m}} \times \frac{d\hat{\boldsymbol{m}}}{dt}\right),$$

$$\text{LL: } \frac{d\hat{\boldsymbol{m}}}{dt} = -\gamma(\hat{\boldsymbol{m}} \times \boldsymbol{H}_{\text{eff}}) - \alpha\gamma\hat{\boldsymbol{m}} \times (\hat{\boldsymbol{m}} \times \boldsymbol{H}_{\text{eff}}),$$

$$\boldsymbol{H}_{\text{eff}} \equiv -\frac{1}{M_s}\frac{\partial E}{\partial \hat{\boldsymbol{m}}} \text{ (variational derivative).} \tag{1}$$

Here, the unit vector $\hat{\boldsymbol{m}}(\boldsymbol{r},t)=\boldsymbol{M}(\boldsymbol{r},t)/M_s$, with the saturation magnetization taken as constant as per the usual micromagnetic approximation. The gyromagnetic ratio γ is taken here to be positive, as is the dimensionless phenomenological damping parameter $\alpha > 0$. As is also well known, the form for G in Eq. (1) may be algebraically rearranged as

$$\text{G: } (1+\alpha^2)\frac{d\hat{\boldsymbol{m}}}{dt} = -\gamma(\hat{\boldsymbol{m}} \times \boldsymbol{H}_{\text{eff}}) - \alpha\gamma\hat{\boldsymbol{m}} \times (\hat{\boldsymbol{m}} \times \boldsymbol{H}_{\text{eff}}), \tag{2}$$

which is *identical* in form to LL excluding terms that are *second* order in damping constant α. It is stipulated here that, in agreement with arguments in Ref. 1, such second-order differences are not readily subject to experimental test, and will not be considered further.

However, the presence of spin-transfer torque in current carrying ferromagnets, whether in ferromagnetic nanowires as considered in Stiles *et al.*[1] or in more general structures, introduces a potential source of *nonconservative* fields $\boldsymbol{H}_{\text{NC}}$ which cannot be expressed as the $\hat{\boldsymbol{m}}$ gradient of an energy functional. Such terms will instead be supposed as derivable from a torque density functional $\boldsymbol{N}(\hat{\boldsymbol{m}})$ by considering the virtual work[4,5] $\delta W_{\text{NC}} \equiv \boldsymbol{H}_{\text{NC}} \cdot \delta\boldsymbol{M}$ done by $\boldsymbol{H}_{\text{NC}}$ during a virtual displacement $\delta\boldsymbol{M}$. Since $|\boldsymbol{M}|=M_s$ is fixed, $\delta\boldsymbol{M}$ is of the form $\delta\boldsymbol{M}=M_s\delta\boldsymbol{\theta}\times\hat{\boldsymbol{m}}$. Further, since $\delta W_{\text{NC}}=\boldsymbol{N}\cdot\delta\boldsymbol{\theta}$ by definition of the torque (density) $\boldsymbol{N}$, it follows that

$$\delta W_{\text{NC}} \equiv \boldsymbol{H}_{\text{NC}} \cdot \delta\boldsymbol{M} = M_s(\hat{\boldsymbol{m}} \times \boldsymbol{H}_{\text{NC}}) \cdot \delta\boldsymbol{\theta}$$
$$\Rightarrow \boldsymbol{N} = M_s(\hat{\boldsymbol{m}} \times \boldsymbol{H}_{\text{NC}}) \Leftrightarrow \boldsymbol{H}_{\text{NC}} = \frac{(\boldsymbol{N} \times \hat{\boldsymbol{m}})}{M_s}. \tag{3}$$

With constant $|\boldsymbol{M}|$, only orthogonal components of the torque $\boldsymbol{N} \leftrightarrow \hat{\boldsymbol{m}} \times \boldsymbol{N} \times \hat{\boldsymbol{m}}$ are physically significant, and $\boldsymbol{H}_{\text{NC}}$ (like $\boldsymbol{H}_{\text{eff}}$) is arbitrary to within a term collinear with $\hat{\boldsymbol{m}}$. For completeness, it is noted that the remaining discussion in Secs. I–III remains intact even for a "conservative" $\boldsymbol{H}_{\text{NC}} = -1/M_s \partial E_{\text{NC}}/\partial\hat{\boldsymbol{m}}$ *provided* that the functional $E_{\text{NC}}(\hat{\boldsymbol{m}})$ (should it exist) is *specifically excluded* from the definition of the magnetic system's total free-energy functional $E(\hat{\boldsymbol{m}})$ that determines $\boldsymbol{H}_{\text{eff}}$. However, the nonconservative nomenclature will be maintained for purposes of discussion.

When including $\boldsymbol{H}_{\text{NC}}$ in a total field $\boldsymbol{H}_{\text{tot}} \equiv \boldsymbol{H}_{\text{eff}}+\boldsymbol{H}_{\text{NC}}$, the generalized nonconservative equations of motion analogous to Eq. (1) are *uniquely* defined for G but *not* for LL:

$$\text{G: } \frac{d\hat{\boldsymbol{m}}}{dt} = -\gamma(\hat{\boldsymbol{m}} \times \boldsymbol{H}_{\text{tot}}) + \alpha\left(\hat{\boldsymbol{m}} \times \frac{d\hat{\boldsymbol{m}}}{dt}\right), \tag{4a}$$

$$\text{LL: } \frac{d\hat{\boldsymbol{m}}}{dt} = -\gamma(\hat{\boldsymbol{m}} \times \boldsymbol{H}_{\text{tot}}) - \alpha\gamma\hat{\boldsymbol{m}} \times (\hat{\boldsymbol{m}} \times \boldsymbol{H}_{\text{eff}}), \tag{4b}$$

$$\text{LLG: } \frac{d\hat{\boldsymbol{m}}}{dt} = -\gamma(\hat{\boldsymbol{m}} \times \boldsymbol{H}_{\text{tot}}) - \alpha\gamma\hat{\boldsymbol{m}} \times (\hat{\boldsymbol{m}} \times \boldsymbol{H}_{\text{tot}}). \tag{4c}$$

The transformation from Eq. (1) to Eq. (2) analogously indicates that Eq. (4c), denoted here as LLG, will retain its usual equivalence to G of Eq. (4a) excluding terms of order α^2. However, based on energetics arguments and considerations of irreversible thermodynamics, it was argued by

Stiles *et al.*[1] that Eq. (4b), which identically retains the original LL damping term, is instead the correct generalization of Landau-Lifshitz in cases when $\boldsymbol{H}_{NC} \neq 0$. If one adopts this view, then it immediately follows that G and LL will *differ* by terms of *first* order in α in the nonconservative case $\boldsymbol{H}_{NC} \neq 0$. It is this more fundamental first-order difference between G and LL which will be examined further below.

II. ENERGY CONSERVATION

If $\hat{\boldsymbol{m}}(t)$ is a solution of G in Eq. (4a), one can derive the following relationships for the time evolution of the energy density by evaluating vector products of form $\boldsymbol{H}\cdot d\hat{\boldsymbol{m}}/dt$ via substitution of $d\hat{\boldsymbol{m}}/dt$ using the right side of Eq. (4a) (along with trivial vector calculus identities):

$$\frac{1}{M_s}\left(\frac{dE}{dt} = \frac{\partial E}{\partial \hat{\boldsymbol{m}}}\cdot\frac{d\hat{\boldsymbol{m}}}{dt}\right) \equiv -\boldsymbol{H}_{\mathrm{eff}}\cdot\frac{d\hat{\boldsymbol{m}}}{dt}$$
$$= \gamma\boldsymbol{H}_{\mathrm{eff}}\cdot(\hat{\boldsymbol{m}}\times\boldsymbol{H}_{\mathrm{NC}}) - \alpha\boldsymbol{H}_{\mathrm{eff}}\cdot\left(\hat{\boldsymbol{m}}\times\frac{d\hat{\boldsymbol{m}}}{dt}\right)$$
$$= -\gamma\boldsymbol{H}_{\mathrm{NC}}\cdot(\hat{\boldsymbol{m}}\times\boldsymbol{H}_{\mathrm{eff}}) - \alpha\boldsymbol{H}_{\mathrm{eff}}\cdot\left(\hat{\boldsymbol{m}}\times\frac{d\hat{\boldsymbol{m}}}{dt}\right), \tag{5a}$$

$$\frac{1}{M_s}\frac{dW_{\mathrm{NC}}}{dt} \equiv \boldsymbol{H}_{\mathrm{NC}}\cdot\frac{d\hat{\boldsymbol{m}}}{dt}$$
$$= -\gamma\boldsymbol{H}_{\mathrm{NC}}\cdot(\hat{\boldsymbol{m}}\times\boldsymbol{H}_{\mathrm{eff}}) + \alpha\boldsymbol{H}_{\mathrm{NC}}\cdot\left(\hat{\boldsymbol{m}}\times\frac{d\hat{\boldsymbol{m}}}{dt}\right), \tag{5b}$$

$$\left|\frac{d\hat{\boldsymbol{m}}}{dt}\right|^2 = \frac{d\hat{\boldsymbol{m}}}{dt}\cdot(-\gamma\hat{\boldsymbol{m}}\times\boldsymbol{H}_{\mathrm{tot}}) = \gamma(\boldsymbol{H}_{\mathrm{eff}}+\boldsymbol{H}_{\mathrm{NC}})\cdot\left(\hat{\boldsymbol{m}}\times\frac{d\hat{\boldsymbol{m}}}{dt}\right). \tag{5c}$$

For simplicity, $\partial E/\partial t$ terms due to *explicit time dependence* of external sources are excluded. Such terms are not present in common examples where sources are turned on suddenly at $t=0$ and then held constant, and the magnetic system $\hat{\boldsymbol{m}}(t\geq 0)$ naturally evolves in time under the influence of these static source fields. Subtracting Eq. (5b) from Eq. (5a), and using Eq. (5c) one finds

$$\mathrm{G:}\ \frac{dE}{dt} = \frac{dW_{\mathrm{NC}}}{dt} - \alpha\frac{M_s}{\gamma}\left|\frac{d\hat{\boldsymbol{m}}}{dt}\right|^2. \tag{6a}$$

Working out the results analogous to Eqs. (5a) and (5b) for the LL and LLG equations, one finds

$$\mathrm{LL:}\ \frac{dE}{dt} = \frac{dW_{\mathrm{NC}}}{dt} - \alpha M_s\gamma(\hat{\boldsymbol{m}}\times\boldsymbol{H}_{\mathrm{eff}})\cdot(\hat{\boldsymbol{m}}\times\boldsymbol{H}_{\mathrm{tot}}), \tag{6b}$$

$$\mathrm{LLG:}\ \frac{dE}{dt} = \frac{dW_{\mathrm{NC}}}{dt} - \alpha M_s\gamma|\hat{\boldsymbol{m}}\times\boldsymbol{H}_{\mathrm{tot}}|^2. \tag{6c}$$

The results of Eq. (6) are a statement of energy conservation. Namely, the rate of change of the internal free-energy density of the magnetic system is the work done on the system by the (external) nonconservative fields $\boldsymbol{H}_{NC}$ minus the loss of energy (to the lattice/thermal environment) due to damping.

The G damping term in Eq. (6a) is manifestly a strictly lossy, negative-definite contribution to dE/dt. This property also holds for LLG but does *not* strictly hold for LL damping except when $\boldsymbol{H}_{NC}=0$. If a tenet for a physically plausible phenomenological damping term is that it be energetically lossy under all circumstances including $\boldsymbol{H}_{NC}\neq 0$, it is incontrovertible that G, but *not* LL, meets this criterion. In this author's view, repeated statements to the direct contrary in Ref. 1 are simply incorrect.

For G or LL, Eqs. (4) and (6) can also be expressed as

$$\frac{d\hat{\boldsymbol{m}}}{dt} = -\gamma[\hat{\boldsymbol{m}}\times(\boldsymbol{H}_{\mathrm{tot}}+\boldsymbol{H}_{\mathrm{damp}})],$$

$$\frac{dE}{dt} = \frac{dW_{\mathrm{NC}}}{dt} + M_s\boldsymbol{H}_{\mathrm{damp}}\cdot\frac{d\hat{\boldsymbol{m}}}{dt},$$

$$\boldsymbol{H}^{\mathrm{G}}_{\mathrm{damp}} \equiv -\left(\frac{\alpha_G}{\gamma}\right)\frac{d\hat{\boldsymbol{m}}}{dt}, \quad \boldsymbol{H}^{\mathrm{LL}}_{\mathrm{damp}} \equiv \alpha_{\mathrm{LL}}(\hat{\boldsymbol{m}}\times\boldsymbol{H}_{\mathrm{eff}}), \tag{7}$$

where $-\boldsymbol{H}_{\mathrm{damp}}\cdot d\hat{\boldsymbol{m}}/dt$ is the rate of *negative* work done on the system by the "damping field" $\boldsymbol{H}_{\mathrm{damp}}$. As was demonstrated by Brown[4] using a Lagrangian mechanics based derivation of the G equations, $\boldsymbol{H}^{\mathrm{G}}_{\mathrm{damp}} \equiv -1/M_s\,\partial\Re/\partial(d\hat{\boldsymbol{m}}/dt)$ can be obtained from a Rayleigh dissipation function[5] $\Re(d\hat{\boldsymbol{m}}/dt)=(\alpha/2\gamma)M_s|d\hat{\boldsymbol{m}}/dt|^2$, where $2\Re$ is by definition the instantaneous damping rate of energy lost due to the *viscous* "friction" represented by $\boldsymbol{H}^{\mathrm{G}}_{\mathrm{damp}} \propto -d\hat{\boldsymbol{m}}/dt$. Hence, the manifestly lossy property of G damping was "built in" from the beginning and, unlike the derivation (5)→(6), does not depend on whether or not $\hat{\boldsymbol{m}}(t)$ is a solution of Eq. (4a).

Over a finite interval of motion from t_1 to t_2, the change $\Delta E=E(\hat{\boldsymbol{m}}_2)-E(\hat{\boldsymbol{m}}_1)$ is, from Eq. (6a),

$$\mathrm{G:}\ \Delta E = M_s\int_{t_1}^{t_2} dt\left[\boldsymbol{H}_{\mathrm{NC}}(\hat{\boldsymbol{m}}) - \frac{\alpha}{\gamma}\frac{d\hat{\boldsymbol{m}}}{dt}\right]\cdot\frac{d\hat{\boldsymbol{m}}}{dt}. \tag{8}$$

If $\boldsymbol{H}_{NC}$ is nonconservative, the work $\Delta W_{NC} = M_s\int_{t_1}^{t_2}(\boldsymbol{H}_{NC}\cdot d\hat{\boldsymbol{m}}/dt)dt$ is path dependent, and hence depends on the motion $\hat{\boldsymbol{m}}(t_1\leq t\leq t_2)$. Since $\hat{\boldsymbol{m}}(t)$ itself depends on α, the $\boldsymbol{H}_{NC}$ term's contribution to ΔE also can vary with α. Regardless, $\Delta E>0$ can *only* result in the case of a *positive* amount of work ΔW_{NC} being done by $\boldsymbol{H}_{NC}$. These results apply equally to situations where one integrates over the spatial distribution of $\hat{\boldsymbol{m}}(\boldsymbol{r},t)$ to evaluate the total system free energy rather than (local) free-energy density. Total time derivatives d/dt may be replaced by partial derivatives $\partial/\partial t$ where appropriate. Similar relations to Eq. (8) for the special case of spin-torque driven closed orbital motion in spin valves is also discussed elsewhere.[6]

III. DOMAIN WALLS IN NANOWIRES

The case of spin-transfer torque current-driven domain wall motion in ferromagnetic nanowires has garnered much recent attention, and is the specific example considered in Stiles *et al.*[1] and references therein. Here, the spin-torque function $\boldsymbol{N}_{\rm ST}(\hat{\boldsymbol{m}})=\boldsymbol{N}_{\rm ST}^{\rm ad}(\hat{\boldsymbol{m}})+\boldsymbol{N}_{\rm ST}^{\rm nad}(\hat{\boldsymbol{m}})$ is taken to have a predominant "adiabatic" component $\boldsymbol{N}_{\rm ST}^{\rm ad}(\hat{\boldsymbol{m}})$ along with a small "nonadiabatic" component $\boldsymbol{N}_{\rm ST}^{\rm nad}(\hat{\boldsymbol{m}})$ described phenomenologically by the relation $\boldsymbol{N}_{\rm ST}^{\rm nad}\equiv-\beta\hat{\boldsymbol{m}}\times\boldsymbol{N}_{\rm ST}^{\rm ad}$, with $\beta\ll 1$. For a narrow nanowire along the $\hat{\boldsymbol{x}}$ axis, with one-dimensional (1D) magnetization $\hat{\boldsymbol{m}}(x)$ and dc electron current density $\boldsymbol{J}_e=J_e\hat{\boldsymbol{x}}$, the torque function $\boldsymbol{N}_{\rm ST}(\hat{\boldsymbol{m}})$ and associated field $\boldsymbol{H}_{\rm ST}(\hat{\boldsymbol{m}})$ [see Eq. (3)] can be expressed as[1]

$$\boldsymbol{N}_{\rm ST}^{\rm ad}(\hat{\boldsymbol{m}})=\left(\frac{\hbar PJ_e}{2e}\right)\left(\frac{d\hat{\boldsymbol{m}}}{dx}\right),$$

$$\boldsymbol{H}_{\rm ST}=-\left(\frac{\hbar PJ_e}{2M_se}\right)\left(\hat{\boldsymbol{m}}\times\frac{d\hat{\boldsymbol{m}}}{dx}+\beta\frac{d\hat{\boldsymbol{m}}}{dx}\right). \tag{9}$$

Dropping terms of order α^2, Eq. (4b) is easily transformed to a Gilbert-like form:

$$\text{LL: }\frac{d\hat{\boldsymbol{m}}}{dt}=-\gamma\hat{\boldsymbol{m}}\times[\boldsymbol{H}_{\rm tot}-\alpha(\hat{\boldsymbol{m}}\times\boldsymbol{H}_{\rm NC})]+\alpha\left(\hat{\boldsymbol{m}}\times\frac{d\hat{\boldsymbol{m}}}{dt}\right), \tag{10}$$

which differs from G in Eq. (4a) by the term $\alpha(\hat{\boldsymbol{m}}\times\boldsymbol{H}_{\rm NC})$. For domain wall motion in nanowires driven by dc electric currents, as described by Eq. (9), the equations of motion become

$$\text{G: }\frac{d\hat{\boldsymbol{m}}}{dt}+v\frac{d\hat{\boldsymbol{m}}}{dx}=-\gamma\hat{\boldsymbol{m}}\times\boldsymbol{H}_{\rm eff}+\alpha\hat{\boldsymbol{m}}\times\left(\frac{d\hat{\boldsymbol{m}}}{dt}+v\frac{\beta}{\alpha}\frac{d\hat{\boldsymbol{m}}}{dx}\right),$$

$$\text{LL: }\frac{d\hat{\boldsymbol{m}}}{dt}+v\frac{d\hat{\boldsymbol{m}}}{dx}=-\gamma\hat{\boldsymbol{m}}\times\boldsymbol{H}_{\rm eff}+\alpha\hat{\boldsymbol{m}}\times\left(\frac{d\hat{\boldsymbol{m}}}{dt}+v\frac{\alpha+\beta}{\alpha}\frac{d\hat{\boldsymbol{m}}}{dx}\right), \tag{11}$$

where $v=\hbar\gamma PJ_e/2M_se$, and terms of order $\alpha\beta$ are dropped for LL. As noted previously,[1,7–9] Eq. (11) permits purely "translational" solutions of the form $\hat{\boldsymbol{m}}(x,t)=\hat{\boldsymbol{m}}_{\rm eq}(x-vt)$ in the special circumstances where $\beta=\alpha$ in the case of G, or $\beta=0$ in the case of LL. Here, $\hat{\boldsymbol{m}}_{\rm eq}(x)$ is the *static*, equilibrium (minimum E) solution of $(\hat{\boldsymbol{m}}_{\rm eq}\times\boldsymbol{H}_{\rm eff})=0$, and $d\hat{\boldsymbol{m}}/dt=-v\,d\hat{\boldsymbol{m}}/dx\to-v\hat{\boldsymbol{m}}'_{\rm eq}(x-vt)$ with $\hat{\boldsymbol{m}}'_{\rm eq}(q)\equiv d\hat{\boldsymbol{m}}_{\rm eq}/dq$. Evaluating $dW_{\rm ST}/dt=M_s\boldsymbol{H}_{\rm ST}\cdot d\hat{\boldsymbol{m}}/dt$ by taking $\boldsymbol{H}_{\rm ST}$ from Eq. (9), one finds that

$$dW_{\rm ST}/dt=-(vM_s/\gamma)\left[\hat{\boldsymbol{m}}\cdot\left(\frac{d\hat{\boldsymbol{m}}}{dx}\times\frac{d\hat{\boldsymbol{m}}}{dt}\right)+\beta\frac{d\hat{\boldsymbol{m}}}{dx}\cdot\frac{d\hat{\boldsymbol{m}}}{dt}\right]\underset{\hat{\boldsymbol{m}}(x,t)=\hat{\boldsymbol{m}}_{\rm eq}(x-vt)}{\longrightarrow}(\beta v^2M_s/\gamma)|\hat{\boldsymbol{m}}'_{\rm eq}(x-vt)|^2. \tag{12}$$

In translational cases where $d\hat{\boldsymbol{m}}/dt$ is *exactly collinear* to $d\hat{\boldsymbol{m}}/dx$, only the nonadiabatic term does work on the $\hat{\boldsymbol{m}}$ system. Otherwise, the adiabatic contribution is in general nonzero. Possible instability of such translational solutions at sufficiently large velocity v, discussed elsewhere,[9] will not be considered further here.

That aside, the energy interpretation of these translational solutions for G or LL is quite different. For G, the positive rate of work $dW_{\rm ST}/dt$ in Eq. (12) when $\beta=\alpha$ exactly balances the negative damping loss as given in Eq. (6a), with the latter always *nonzero* and scaling as v^2. For LL by contrast, the work done by $\boldsymbol{H}_{\rm ST}$ vanishes when $\beta=0$, matching the damping loss which, from Eq. (6b), is *always zero* since $(\hat{\boldsymbol{m}}_{\rm eq}\times\boldsymbol{H}_{\rm eff})=0$ *regardless* of v. If $\hat{\boldsymbol{m}}_{\rm eq}(x)$ is a sharp domain wall, $d\hat{\boldsymbol{m}}/dt=-v\hat{\boldsymbol{m}}'_{\rm eq}(x-vt)$ represents, from a spatially *local* perspective at a *fixed* point x, an abrupt, irreversible reorientation of $\hat{\boldsymbol{m}}$ at/near time $t\approx x/v$ when the wall core passes by. The interpretation with LL from Eq. (4b) is that of a purely lossless domain wall motion, in which the aforementioned local magnetization reversal can take place with the complete absence of the spin-orbit coupled, electron-scattering processes[8–10] that lead to spin-lattice damping/relaxation in all other known circumstances, e.g., external field-driven domain wall motion. These relaxation processes are the basic physical mechanism behind the phenomenological damping constant α for either G or LL. Thus, although Ref. 1 confirms the mathematical existence of three-dimensional (3D) translational solutions in numerical simulations using LL with $\beta=0$, the seeming nonphysical attributes of such solutions would in the author's view argue in favor of G along with $\beta\approx\alpha$ for interpreting actual experimental observations of current-driven domain wall motion. Theoretical arguments relating β and α are considered in more detail elsewhere.[7–9]

Stiles *et al.*[1] also computed 3D micromagnetic simulations of current-driven domain wall motion using Gilbert equations in the purely adiabatic case $\beta=0$. After the current is turned on, they report nontranslational, time/distance limited domain wall displacement, resulting in a final stationary state with a net *positive* increase ΔE. It is claimed in Ref. 1 that "spin-transfer torques do not change the energy of the system," and that "Gilbert damping torque is solely responsible for this increase in energy." This seemingly unphysical antidissipation property is used as Ref. 1's primary argument against Gilbert. However, it is this author's view that the general results of Eqs. (6a) and (8) show that both of these claims must be incorrect, and that any *increase* in free energy after the current is turned on cannot be attributed to Gilbert damping torque and must be attributable to work done by the spin-transfer torques.

IV. CONSERVATIVE VERSUS NONCONSERVATIVE

Although the question of whether $\boldsymbol{H}_{\rm ST}$ is strictly nonconservative or not is unessential to the earlier results, in this issue it is perhaps worthy of some further comment. A nonconservative torque $\boldsymbol{N}_{\rm NC}$ and/or field $\boldsymbol{H}_{\rm NC}$ simply implies that $\Delta W_{\rm NC}=M_s\int^t(\boldsymbol{H}_{\rm NC}\cdot d\hat{\boldsymbol{m}}/dt')dt'$ is path/history dependent so there exists no state function $W_{\rm NC}(\hat{\boldsymbol{m}})$, and thus $\boldsymbol{H}_{\rm NC}\neq\partial W_{\rm NC}/\partial\hat{\boldsymbol{m}}$. A prototypical example of this is the case of spin torque in a current carrying spin-valve stack essentially comprised of two ferromagnetic layers sandwiching a nonmagnetic conductive spacer. As described by Slonczewski,[11] the (adiabatic) spin torque on either magnetic layer $(i=1,2)$ is $\boldsymbol{N}_i^{\rm ST}\propto J_e\hat{\boldsymbol{m}}_i\times(\hat{\boldsymbol{m}}_2\times\hat{\boldsymbol{m}}_1)$. The field $\boldsymbol{H}_i^{\rm ST}\propto J_e\hat{\boldsymbol{m}}_2\times\hat{\boldsymbol{m}}_1$ [from Eq. (3)] is manifestly nonconservative due to its cross-product form. For the nonadiabatic term $\boldsymbol{N}_{\rm ST}^{\rm nad}=-\beta\hat{\boldsymbol{m}}\times\boldsymbol{N}_{\rm ST}^{\rm ad}$, it follows for the spin-valve case that $\boldsymbol{H}_{i=1,2}^{\rm nad}\propto(2i-3)\beta J_e\partial(\hat{\boldsymbol{m}}_1\cdot\hat{\boldsymbol{m}}_2)/\partial\hat{\boldsymbol{m}}_i$ is "operationally conservative" in the common "one-layer" model, where only *one* of $\hat{\boldsymbol{m}}_1$ or $\hat{\boldsymbol{m}}_2$ is treated as "the system" and the other as a fixed parameter, but nonconservative for the more general two-layer model due to the $(2i-3=\pm 1)$ factor.

A systematic conservative/nonconservative criterion can be defined within the standard micromagnetic discretization approximation whereby the continuum $\hat{\boldsymbol{m}}(\boldsymbol{r})$ is replaced by discrete, uniformly magnetized "cells:" the ith cell magnetization $\hat{\boldsymbol{m}}_i \equiv \hat{\boldsymbol{m}}(\boldsymbol{r}_i)$. In particular, for the nanowire case described by Eq. (9), $d\hat{\boldsymbol{m}}/dx|_{x=x_i} \rightarrow (\hat{\boldsymbol{m}}_{i+1} - \hat{\boldsymbol{m}}_{i-1})/2\Delta x$, and $\Delta x \equiv x_{i+1} - x_i$. A *conservative* $\boldsymbol{H}_i^{\mathrm{ST}}$, such that $\boldsymbol{H}_i^{\mathrm{ST}} \propto -\partial E_{\mathrm{ST}}/\partial \boldsymbol{m}_i$, will yield a *symmetric* matrix of 3D Cartesian tensors $\overleftrightarrow{H}_{ij}^{uv} \equiv -\partial H_i^u/\partial m_j^v \propto \partial^2 E_{\mathrm{ST}}/\partial m_i^u \partial m_j^v$, such that $\overleftrightarrow{H}_{ij}^{uv} = \overleftrightarrow{H}_{ji}^{vu}$ under simultaneous reversal of *both* spatial indices i, j, and vector indices u, $v = x$, y, or z. For the discretized nonadiabatic term in Eq. (9), it is readily shown that the $\hat{\boldsymbol{m}}$-independent $\overleftrightarrow{H}_{ij}^{uv}$ is always antisymmetric, i.e., symmetric in vector indices, but antisymmetric in spatial indices $i, j = i, i\pm 1$. However, for the adiabatic term in Eq. (9), it is straightforwardly shown that $\overleftrightarrow{H}_{ij}^{uv}$ is linear in $\hat{\boldsymbol{m}}$ and in general *asymmetric*, i.e., always antisymmetric in vector indices (due to cross product), but generally asymmetric in spatial indices $i, j = i, i\pm 1$. The only exception is for (locally) *uniform magnetization* $\hat{\boldsymbol{m}}_{i\pm 1} = \hat{\boldsymbol{m}}_i$, in which case, $\overleftrightarrow{H}_{ij}^{uv}$ becomes pure antisymmetric in spatial indices, and thus symmetric overall. In the continuum limit $\Delta x \rightarrow 0$, this uniformity criterion might appear to be met asymptotically, which may reconcile with very different, continuum-limit mathematical arguments[12] asserting that $\boldsymbol{N}_{\mathrm{ad}}$ for the nanowire is conservative. However, for discretized micromagnetic simulations such as those carried out by Stiles *et al.*,[1] $\boldsymbol{N}_{\mathrm{ad}}$ would nonetheless appear to be nonconservative in an operational sense. Grain boundaries/defects could potentially play a similar discretization role in physical magnetic nanowires. For the spin-valve case in which the layers and interfaces are naturally discrete, this ambiguity does not arise.

V. SUMMARY

It is shown that the Gilbert damping term $\alpha\ (\hat{\boldsymbol{m}} \times d\hat{\boldsymbol{m}}/dt)$ in the micromagnetic equations of motion retains its physically intuitive, purely energy dissipative property in the additional presence of nonconservative fields $\boldsymbol{H}_{\mathrm{NC}}$, such as is physically realized in situations of present day interest where spin-transfer torques are present. Contrary to prior findings,[1] this physical property is shown to *not* be rigorously met by the conventional Landau-Lifshitz damping term $-\alpha\gamma\hat{\boldsymbol{m}} \times (\hat{\boldsymbol{m}} \times \boldsymbol{H}_{\mathrm{eff}})$, the use of which was also argued here to lead to other seemingly nonphysical results (to first order in α) in cases where $\boldsymbol{H}_{\mathrm{NC}} \neq 0$. This apparent deficiency can be remedied by replacing $\boldsymbol{H}_{\mathrm{eff}} \rightarrow \boldsymbol{H}_{\mathrm{eff}} + \boldsymbol{H}_{\mathrm{NC}}$ in the Landau-Lifshitz form.

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